Масштабная инвариантность в задаче двумерного случайного роста

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Two dimensional aggregate growth

Ice crystals

D_3=2.2-2.6

D_2=1.4-1.8

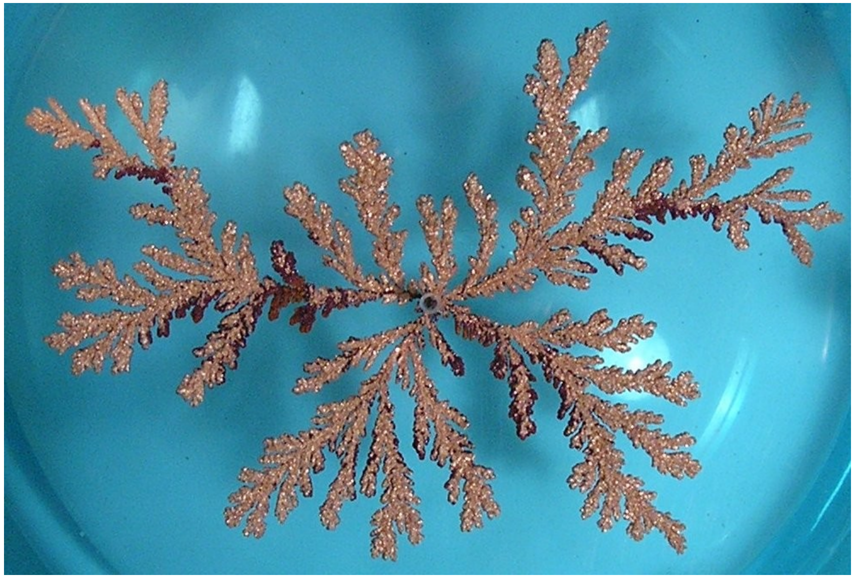
Two dimensional aggregate growth

Natural Cu

Goethite in agate

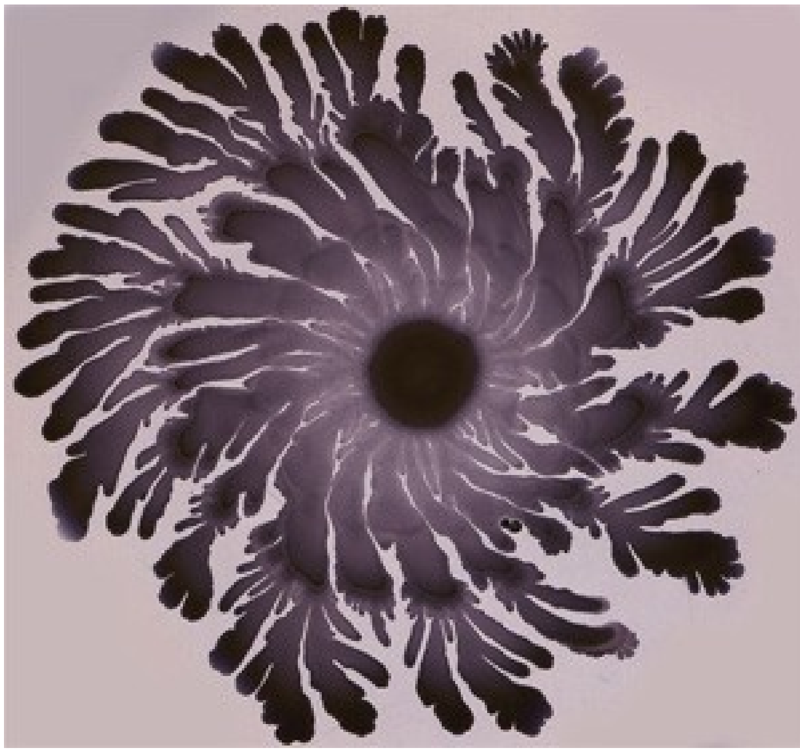
D=1.5-1.8

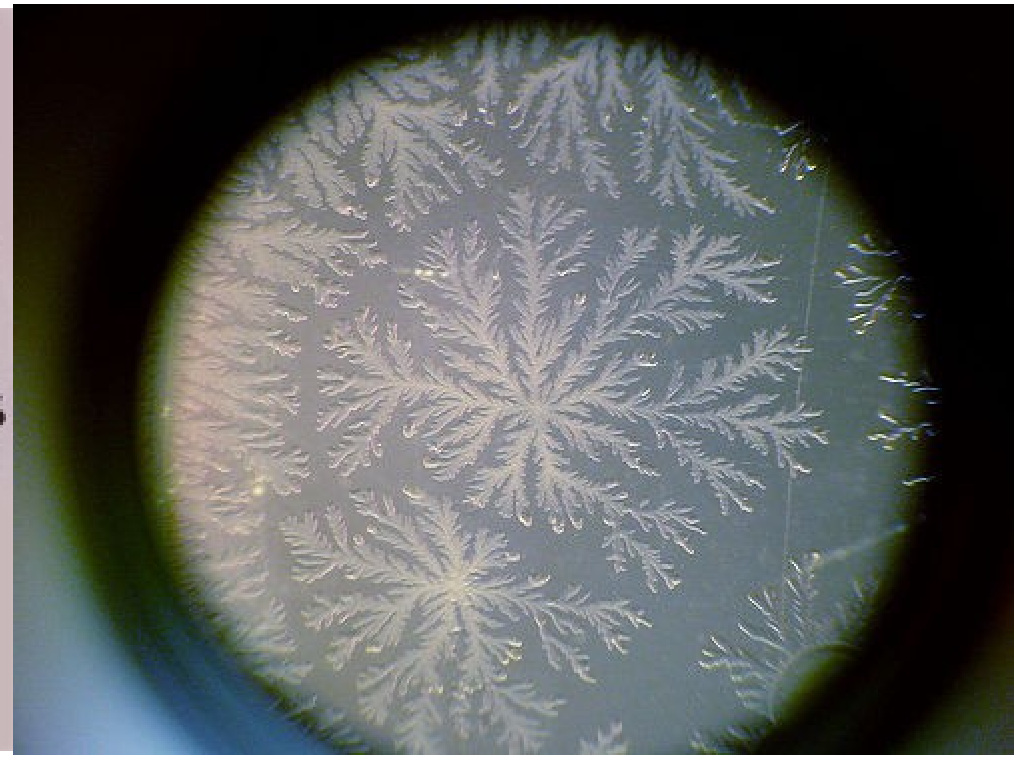
Dendrits & aggregates



Manganese oxide in chalcedony

Two dimensional aggregate growth





Bacteria colonia Bacillus subtilis
from the site www.igmors.u-psud.fr

D=1.7

Two dimensional aggregate growth

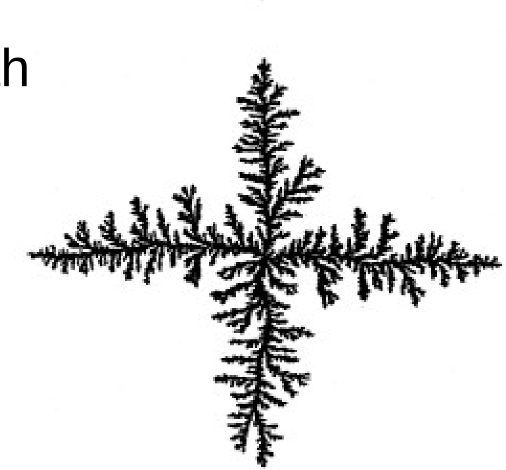
D=1.6-1.9

Nano-crystals grown on a substrate

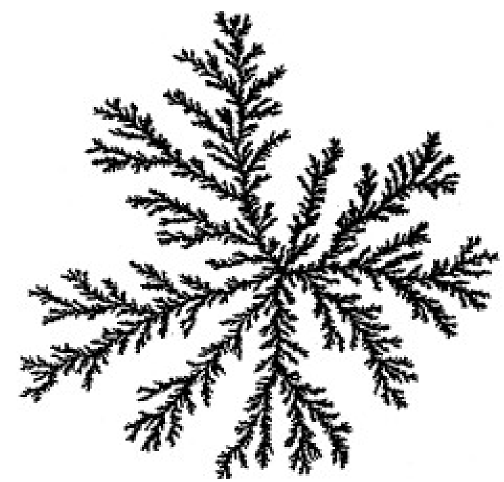
A model for two-dimensional aggregate growth — DLA model

Diffusion limited aggregation – DLA
Witten and Sander, PRL, 1981

1. Place seed at origin (0,0), N=1
2. Particle starts at radius of birth Rbirth
3. Diffusion in space
4. If collision occurs, it sticks. N=N+1
5. If particles goes out of the radius of death R_{death} it is killed
6. New iteration – from step 2.



D=3/2 (?)
on the square lattice



D=1.72
Off-lattice

Problem definition

- Large class of models of 2D growth results in similar structures (weakly speaking - “*looks similar*”)
- DLA model is the simplified model that captures main features of 2D critical aggregate growth
- Scaling exponents are known only with 1% accuracy
- Cluster behavior as $N \to \infty$ is still not understood
- Multiscaling?

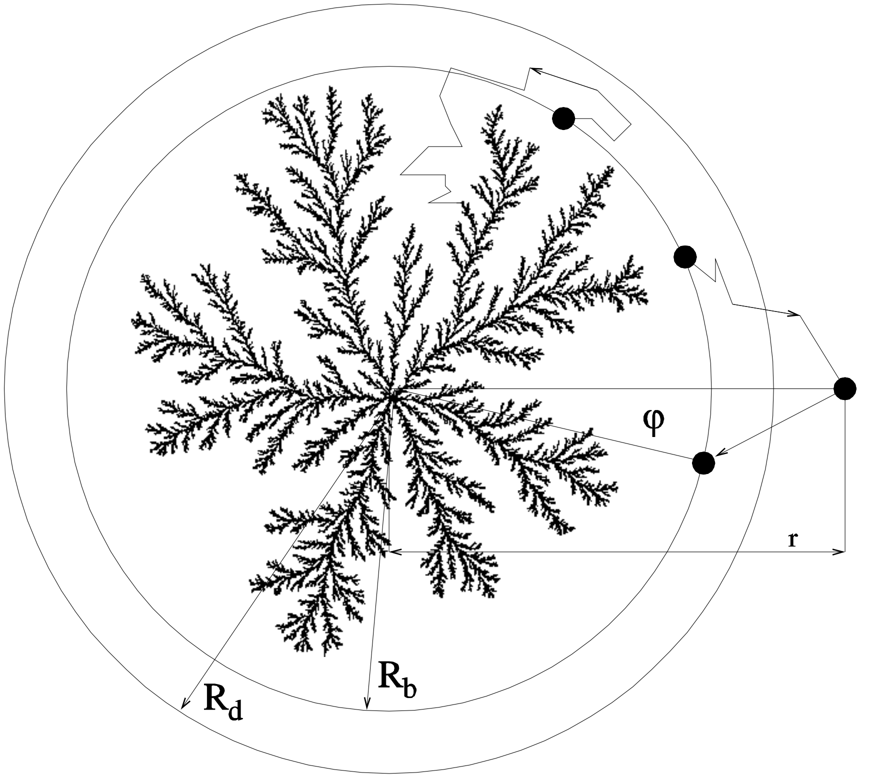
Off-lattice killing-free algorithm

The most effective and most correct realization of algorithm

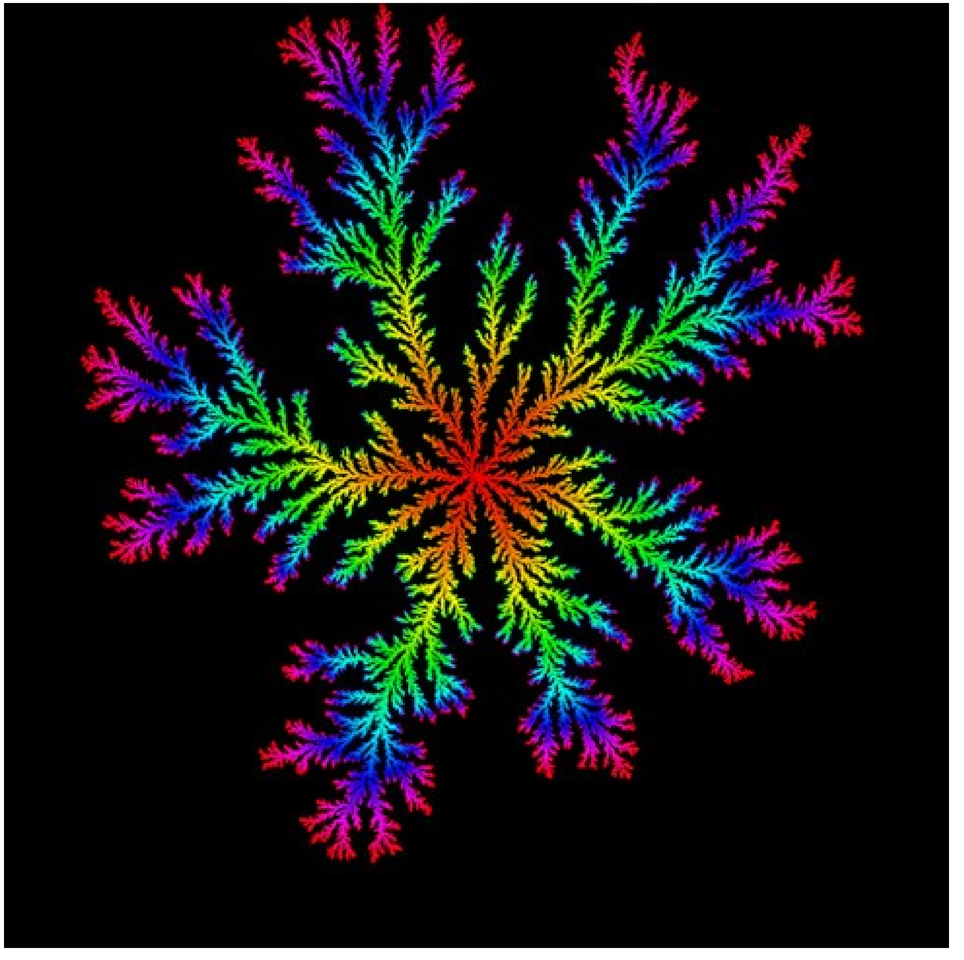
Particles are balls of unit radius.
A single particle at (0,0) position - a seed.
At random point on birth circle Rb a new particle is born.
Particle moves randomly.
If particle goes out of Rd (Rd>Rd) it is returned to Rb at angle phi with probability

$$W(\varphi) = \frac{1}{2\pi} \frac{x^2 - 1}{x^2 - 2x\cos\varphi + 1} \quad x = r / R_b$$

Repeat random walk until collision with cluster.



Off-lattice killing-*free* algorithm



50 000 000 particles

1000 clusters in each ensemble

Scaling in DLA

$$N \propto R^D$$

How to measure R?
Take an average over cluster surface with weight dq
(growth probability at point r)

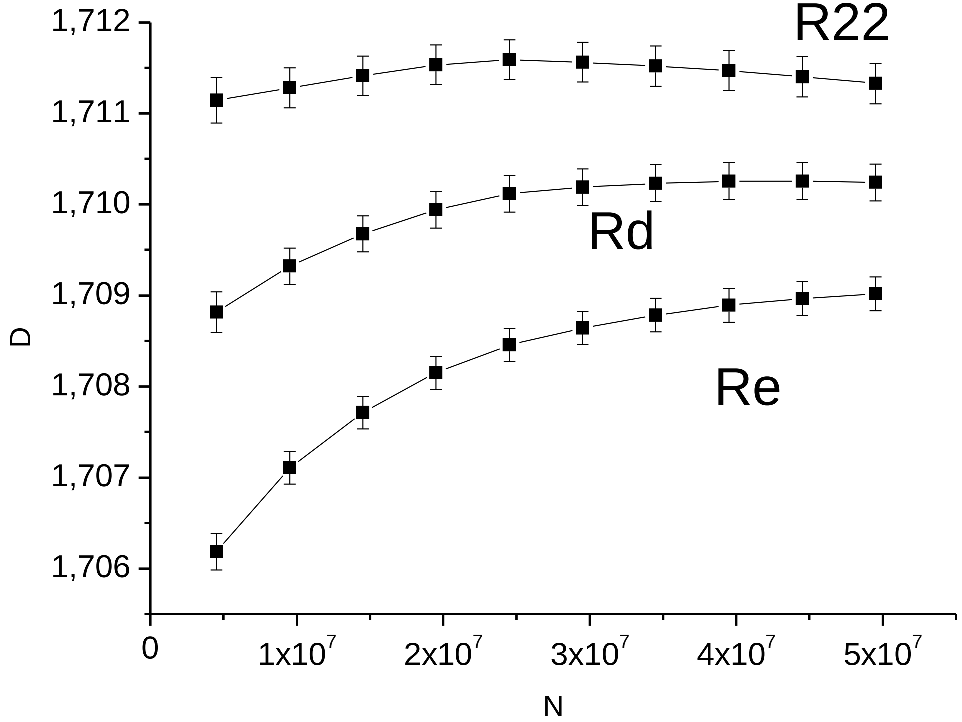
$$R_{dep} = \int r\, dq$$

$$R_{22} = \sqrt{\int r^2\, dq}$$

$$R_e = \exp \int \ln(r)\, dq$$

Numerically: freeze cluster, launch probe particles, record their position r_i and average it.

$$R_{dep} = \frac{1}{M} \sum_1^M r_i$$



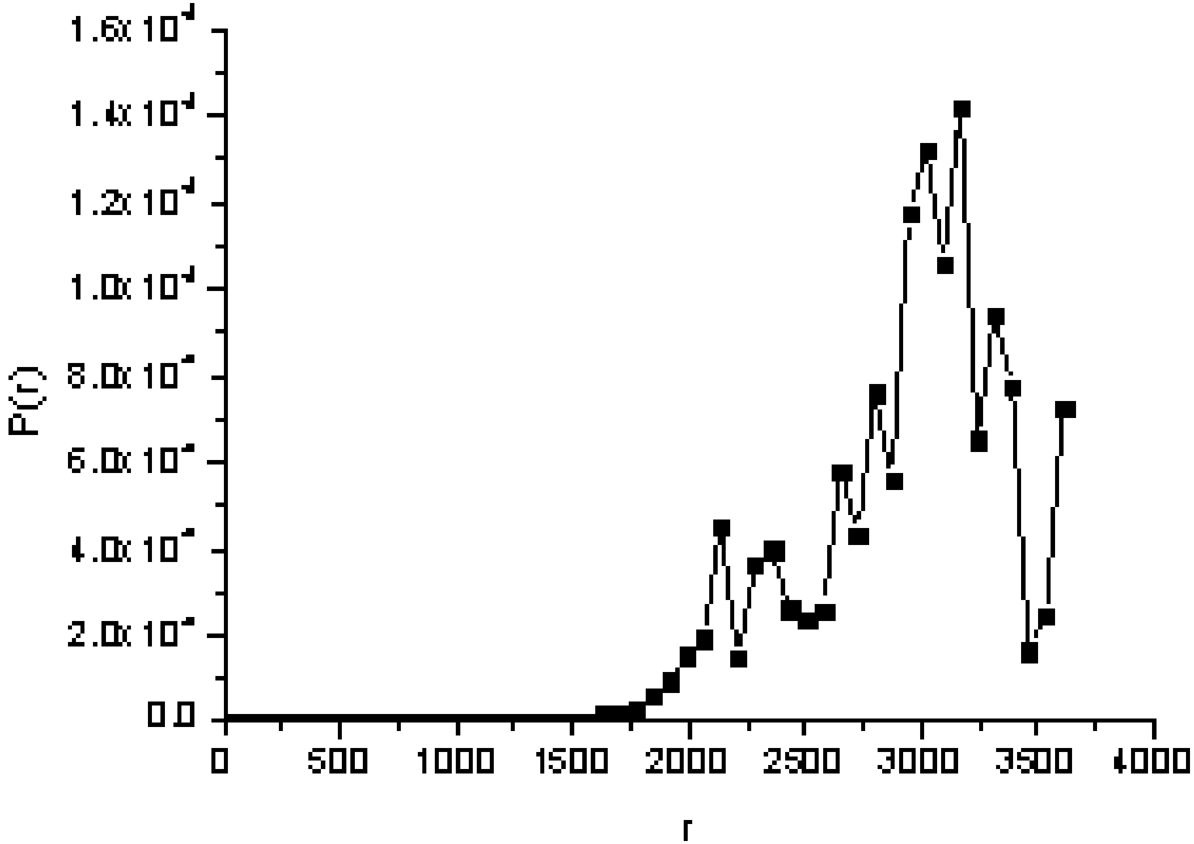
Limit

$N \to \infty$

?????

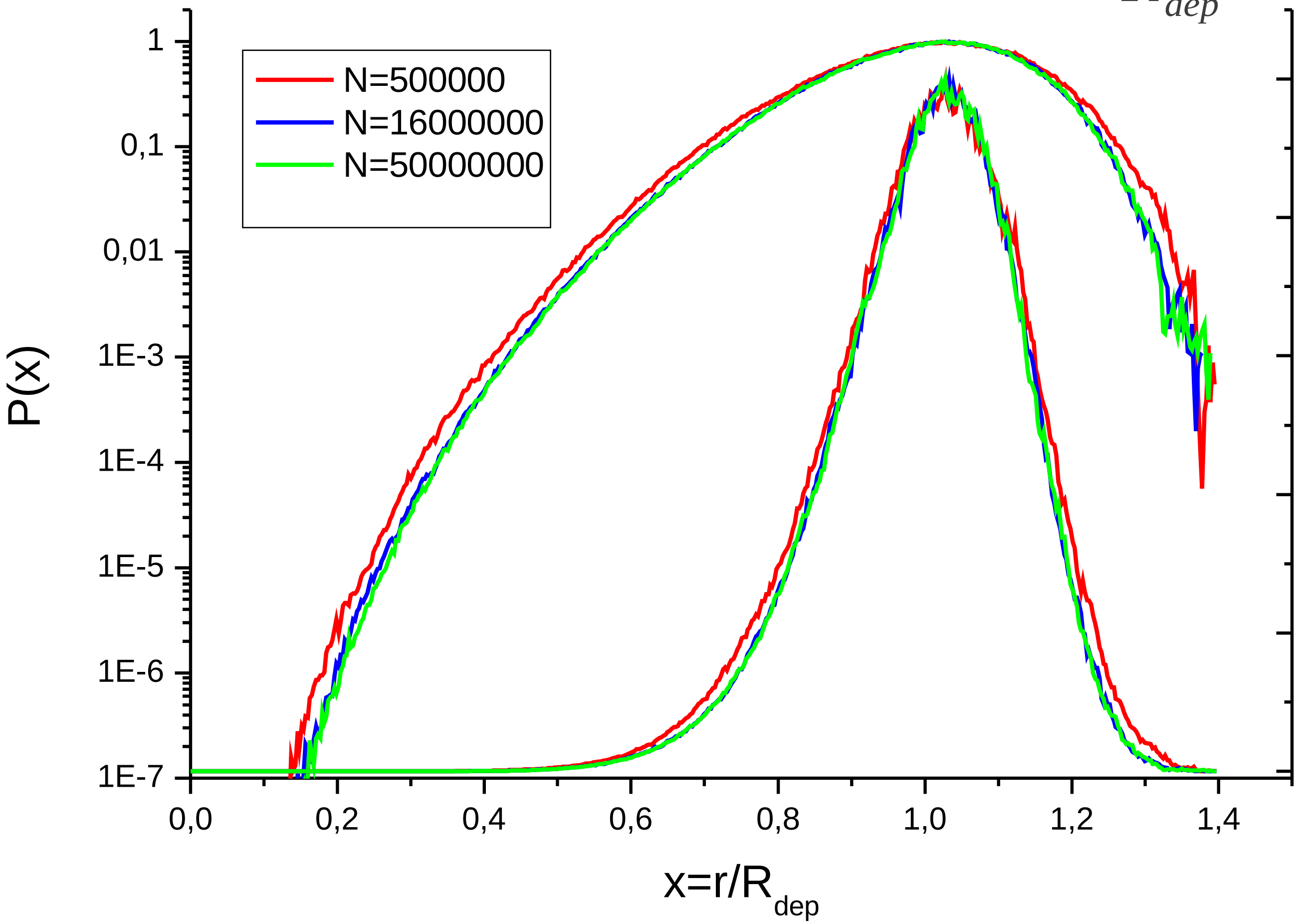
Growth probability

Let P(r,N) be a probability for a next particle to be attached at distance r and N is a size of cluster.



Growth probability

Let us assume a scale invariant form of P(r,N) $P(r,N) = \frac{1}{R_{dep}} f(r/R_{dep})$



Scaling assumption and consequences

$$P(r,N)=\frac{1}{R_{dep}}\,f(r/R_{dep})$$ <=>

Scaling indexes of Rdep, R22, Re ... are equal.

$$R_{dep}=\int r\,P(r,N)\,dr \qquad R_e=\exp\int \ln(r)\,P(r,N)\,dr$$

$$R_{22}=\sqrt{\int r^2 P(r,N)\,dr}$$

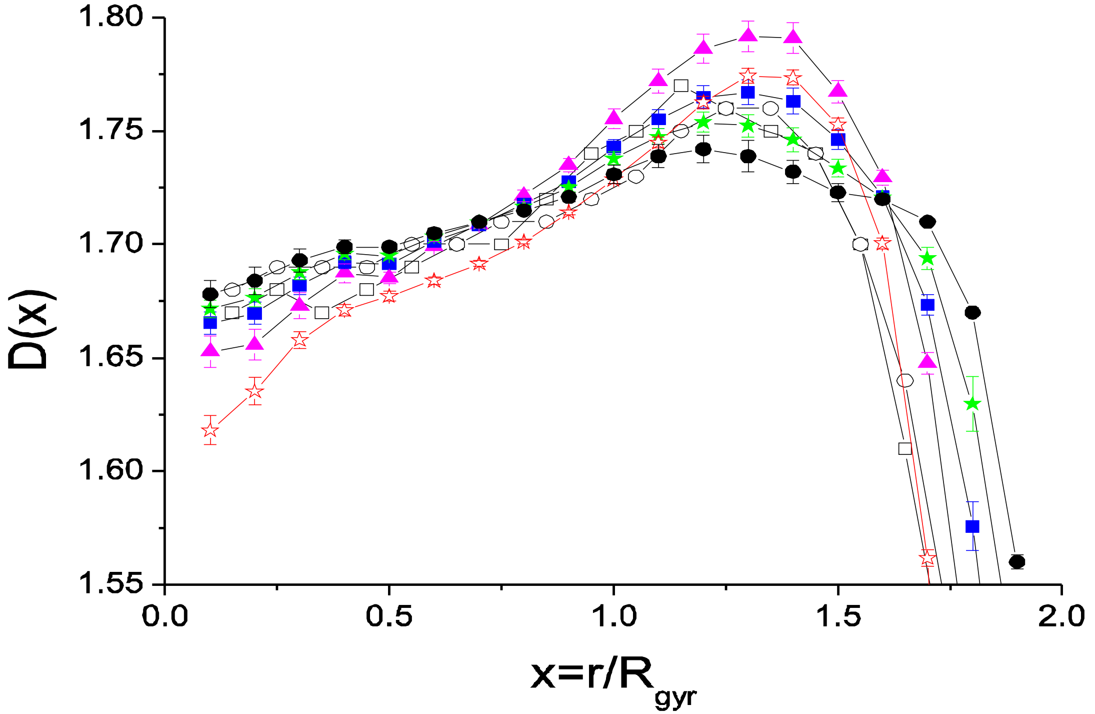
$$R_{dep}\propto R_{22}\propto R_e$$

Multiscaling in DLA

Particle density at distance r from cluster origin

$$g(r, N) = A(x) R_{gyr}^{D(x)-2}, x = r / R_{gyr}$$

Non-trivial family of indexes D(y)?



Scaling assumption and consequences

No multiscaling

$$R_{dep} \propto N^{1/D}$$

+

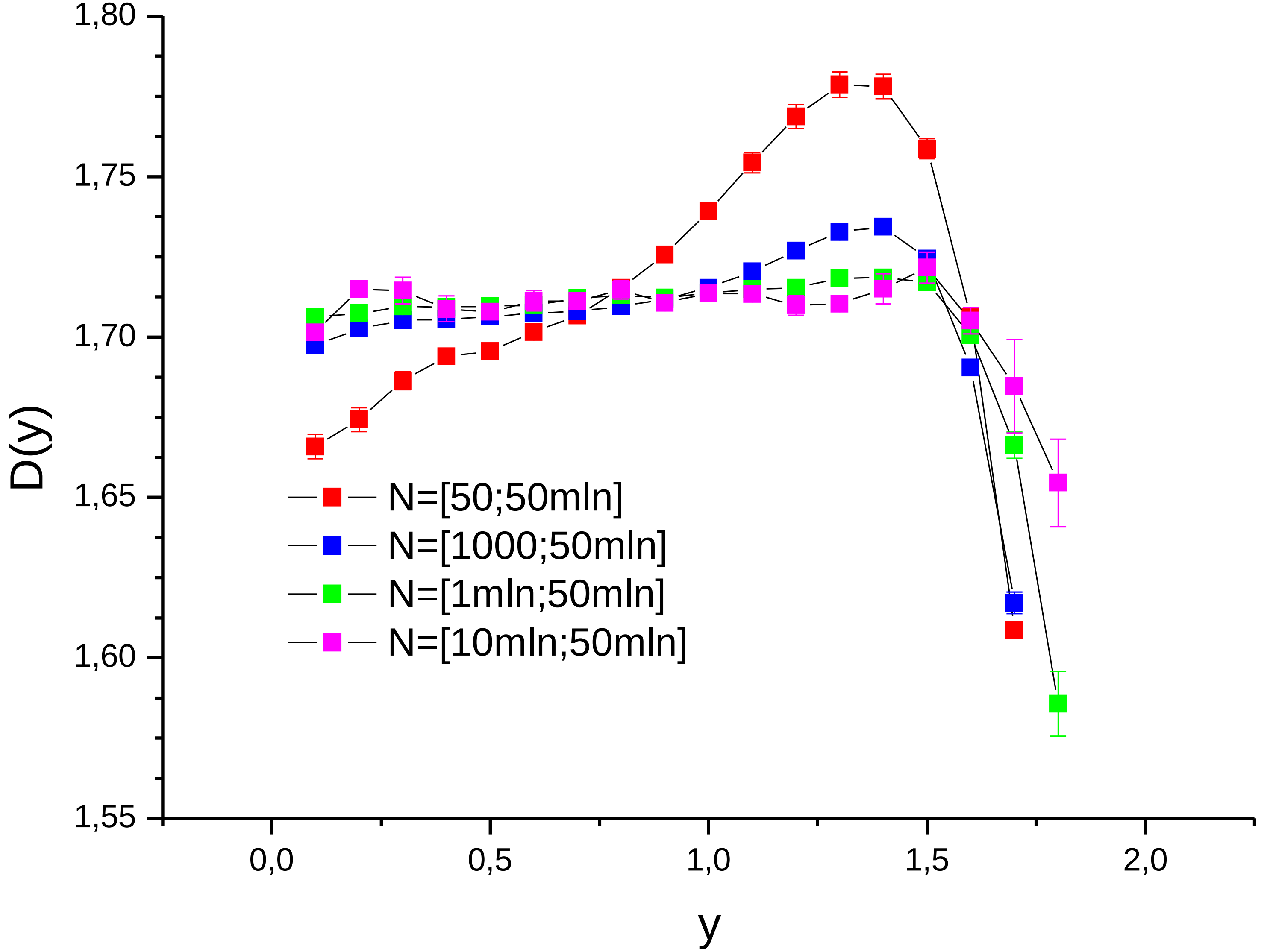
$$g(r,N) = \int_{K=1}^{K=N} P(r,K)\,dK$$

$$g(r,N) \propto r^{(D-2)} F(r/R_{dep}(N))$$

$$F(t) = \int_{t}^{\infty} f(x)\,x^{-D}\,dx$$

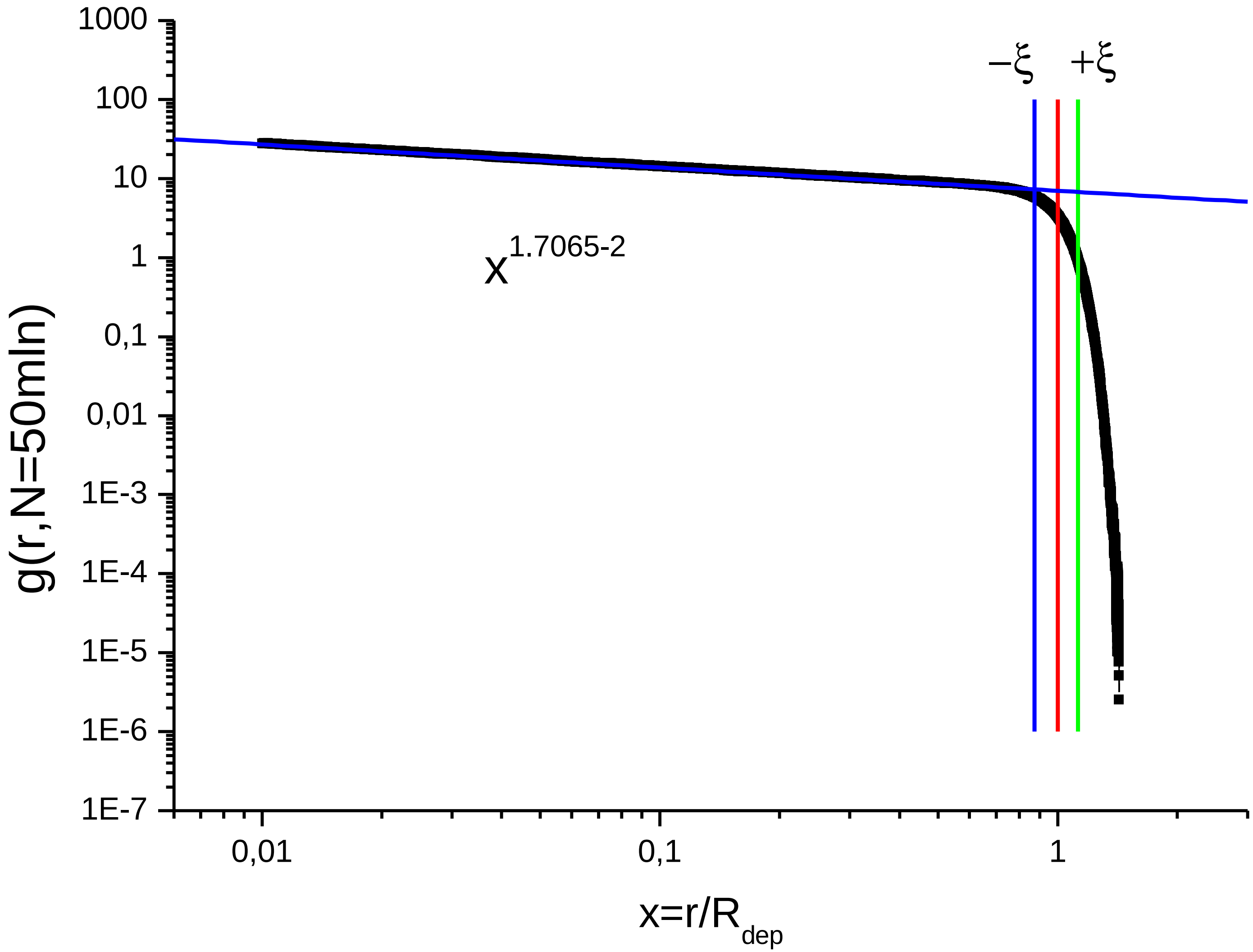
Numerical check of multiscaling

$$\ln g(r, R_{gyr}) = \ln c(y) + (D(y) - 2)\ln R_{gyr}$$



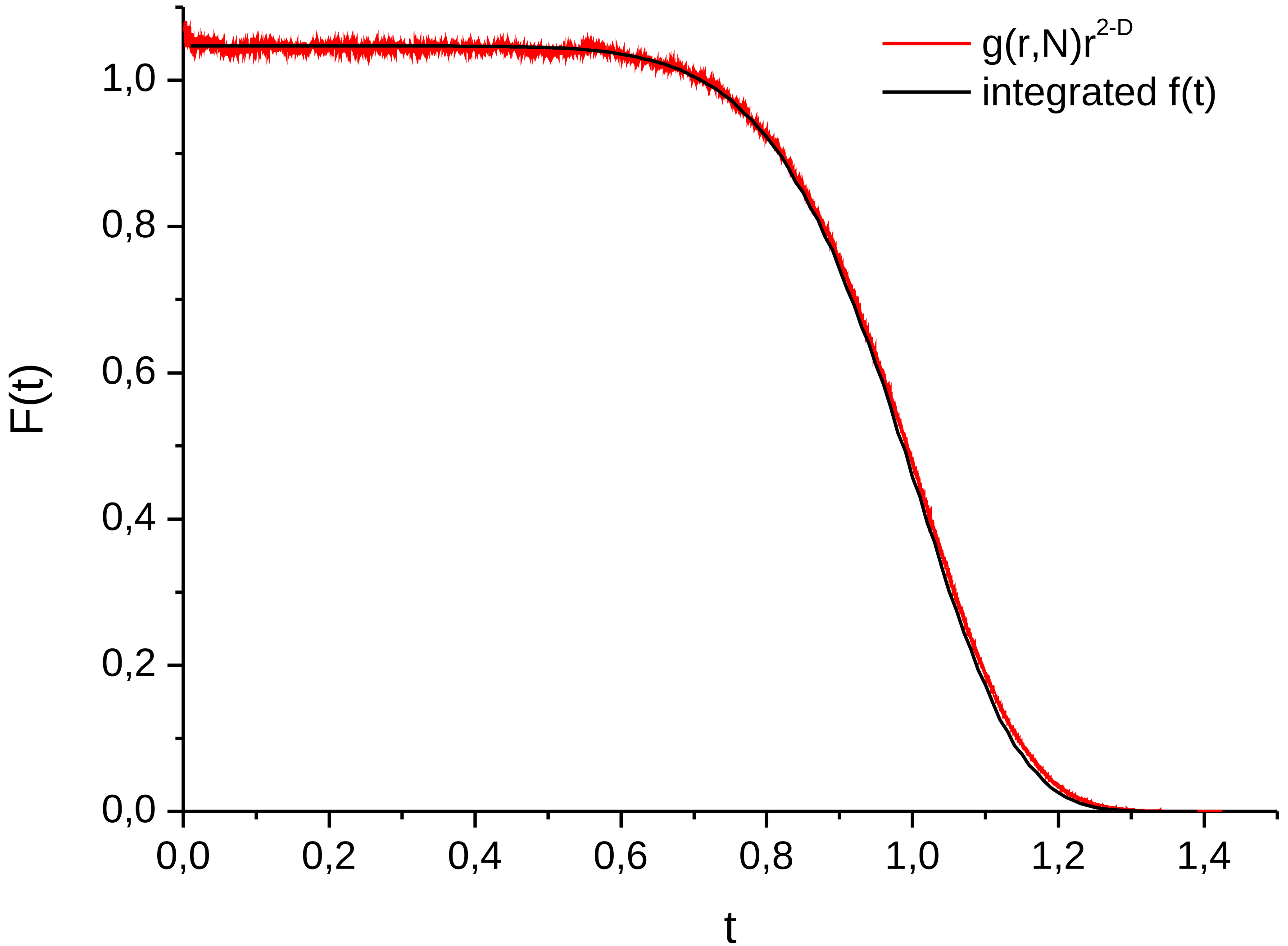
Radial density in scale invariant form

$$g(r,N) \propto x^{(D-2)} F(r/R_{dep}(N)),\ x = r/R_{dep}$$



Scaling function F(t)

$$F(t) = \int_t^{\infty} f(x)\, x^{-D}\, dx \qquad f(x)=P(r,N=50mln)$$



Conclusion

1) New scale-invariant form of growth probability P(r,N) suggested.
2) “All” scaling exponents are equal to D.
3) Multiscaling **is finite size effect.**
4) Density of particles $g(r,N) \propto x^{(D-2)} F(r/R_{dep}(N)), x = r/R_{dep}$
5) No collapse as $N \to \infty$
6) DLA is a model of Self-organized criticality class.

Suggestion:
Need to derive P(r,N) on g(r,N) dependence for exact P(r,N) calculation.