Dynamic properties of one class of impulse systems

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Dynamic system

$$\dot{u}_j = d(u_{j+1} - 2u_j + u_{j-1}) + \lambda[-1 + \alpha f(u_j(t-1)) - \beta g(u_j)]u_j, \quad j = \overline{1,m}, \qquad (1)$$

$$\underline{u_0 = u_1,\ u_m = u_{m+1},}$$

$$u_j = u_j(t) > 0,\ m \geq 2,\ \lambda >> 1,\ \beta > 0,\ \alpha > 1 + \beta,$$

$$f(u), g(u) \in C^2(\mathbb{R}_+): \quad \mathbb{R}_+ = \{u \in \mathbb{R} : u \geqslant 0\},$$

$$0 < \beta g(u) < \alpha,\ f(0) = g(0) = 1, \quad \forall u \in \mathbb{R}_+;$$

$$f(u), g(u), uf'(u), ug'(u), u^2 f''(u), u^2 g''(u) = O(1/u), \quad u \to +\infty.$$

Glyzin S.D., Kolesov A.Yu, Rozov N.Kh. Relaxation self-oscillations in neuron systems. III // Differential Equations. 2012. V. 48, № 2. P. 159 – 175.

Substitutions

$$u_1 = \exp\left(\frac{x}{\varepsilon}\right), \quad \varepsilon = \frac{1}{\lambda} << 1.$$

$$u_j = \exp\left(\frac{x}{\varepsilon} + \sum_{k=1}^{j-1} y_k\right), \quad j = \overline{2, m},$$

$$\dot{x} = -1 + \alpha f\left(\exp\left(\frac{x(t-1)}{\varepsilon}\right)\right) - \beta g\left(\exp\left(\frac{x}{\varepsilon}\right)\right)$$

Glyzin S.D., Kolesov A.Yu, Rozov N.Kh. Relaxation self-oscillations in neuron systems. III // Differential Equations. 2012. V. 48, № 2. P. 159 – 175.

Limit impulse system

$$\dot{y}_j = d[\exp y_{j+1} + \exp(-y_j) - \exp y_j - \exp(-y_{j-1})], \quad j = \overline{1, m-1}, \tag{2}$$

$$y_j(+0) = \frac{\alpha - 1}{\alpha - \beta - 1} y_j(-0),$$

$$y_j(1+0) = y_j(1-0) - \frac{\alpha}{\alpha - 1} y_j(+0),$$

$$y_j(\alpha + 0) = (1 + \beta) y_j(\alpha - 0),$$

$$y_j(\alpha + 1 + 0) = y_j(\alpha + 1 - 0) - \frac{\alpha}{1 + \beta} y_j(\alpha + 0),$$

$$y_0 = y_m = -\sum_{k=1}^{m-1} y_k.$$

Research mapping

$$\Phi(z) : \begin{pmatrix} z_1 \\ \vdots \\ z_{m-1} \end{pmatrix} \to \begin{pmatrix} y_1(T_0) \\ \vdots \\ y_{m-1}(T_0) \end{pmatrix}, \qquad (3)$$

$$y_1(-0) = z_1, \ \ldots \ , \ y_{m-1}(-0) = z_{m-1}$$

$$T_0 = \alpha + 1 + (\beta + 1)/(\alpha - \beta - 1)$$

Theorem: For any stable point z_* of mapping (3) there exists a stable relaxational cycle with period T_0 in system (1).

Method of research

$$\Phi(z) : \begin{pmatrix} z_1 \\ \vdots \\ z_{m-1} \end{pmatrix} \to \begin{pmatrix} y_1(T_0) \\ \vdots \\ y_{m-1}(T_0) \end{pmatrix},$$

$$y_1(-0) = z_1, \ \ldots \ , y_{m-1}(-0) = z_{m-1}$$

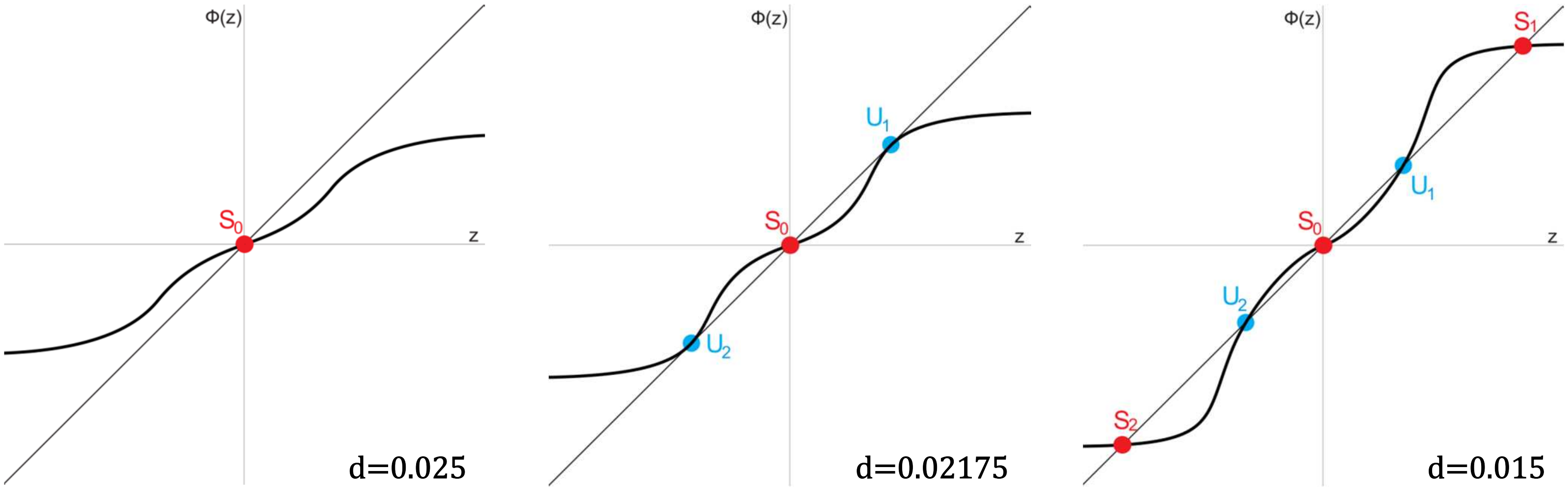
$$T_0 = \alpha + 1 + (\beta + 1)/(\alpha - \beta - 1)$$

Results of research: one-dimensional case

$$m = 2$$

$$\dot{y} = d(e^{-y} - e^{y})$$

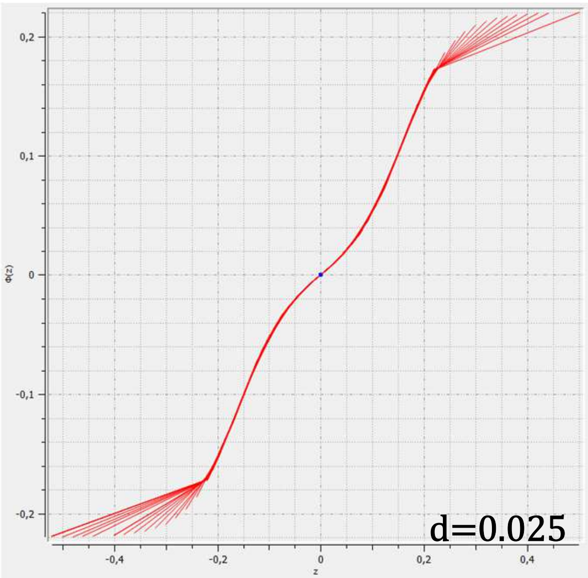
$$\Phi(z) : z \to y(T_0)$$

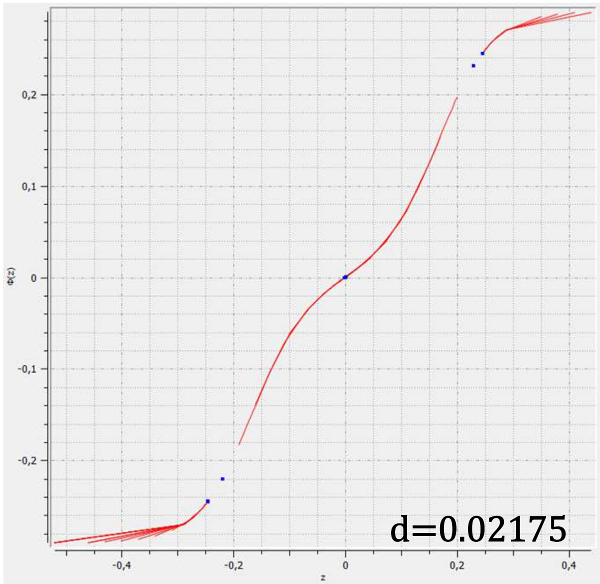


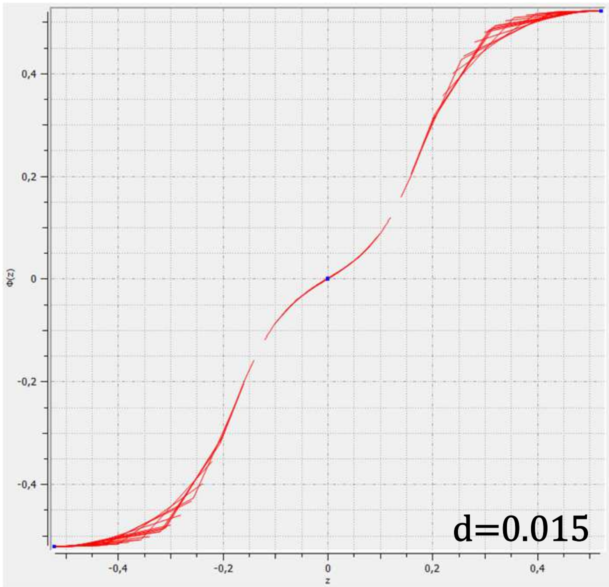
$$\alpha = 1.1, \quad \beta = 0.05$$

Glyzin S.D., Kolesov A.Yu, Rozov N.Kh. Relaxation self-oscillations in neuron systems. III // Differential Equations. 2012. V. 48, № 2. P. 159 – 175.

Screenshots of program for one-dimensional-case







$$\alpha = 1.1, \quad \beta = 0.05$$

Results of research: two-dimensional case

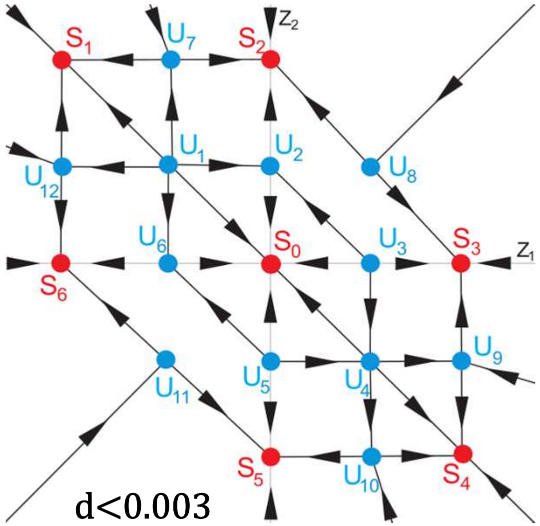
$$m = 3$$

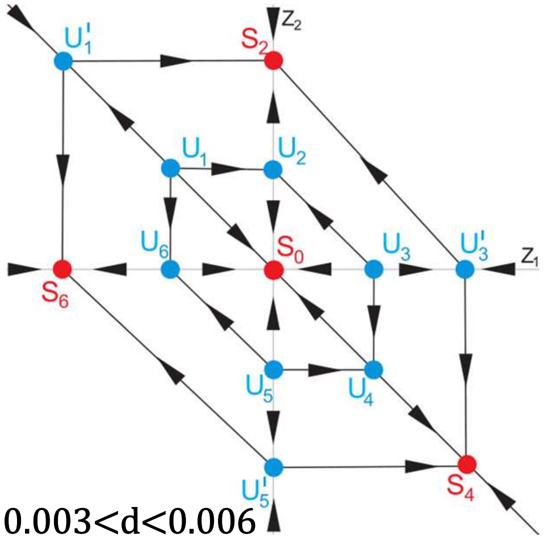
$$\begin{cases} \dot{y_1} = d(e^{y_2} + e^{-y_1} - e^{y_1} - 1) \\ \dot{y_2} = d(1 + e^{-y_2} - e^{y_2} - e^{-y_1}) \end{cases}$$

$$\Phi(z) : \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \to \begin{pmatrix} y_1(T_0) \\ y_2(T_0) \end{pmatrix}$$

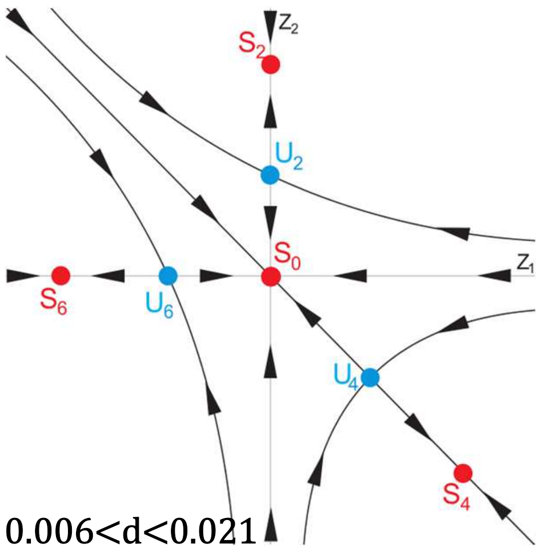
Ivanovsky L., Samsonov S. Phase reconstructions of two-dimensional dynamic system with impulsive influences // Modelling and Analysis of Information Systems , 2014. V. 21, №6. P. 179 – 181.

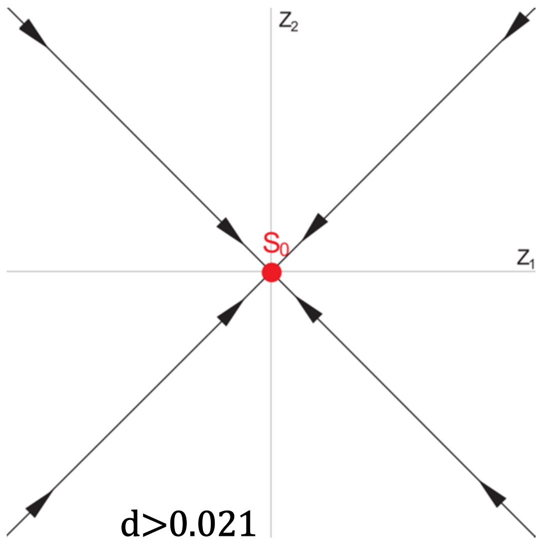
Bifurcations





$\alpha = 1.9, \quad \beta = 0.1$





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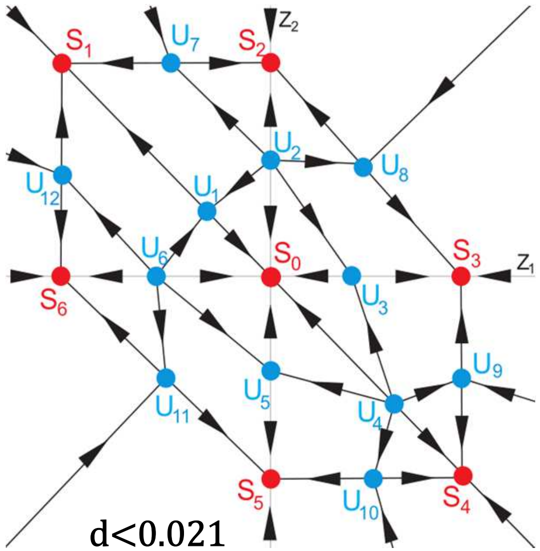
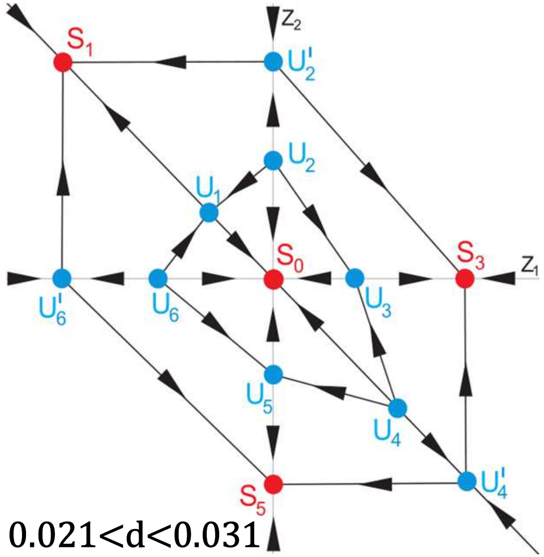
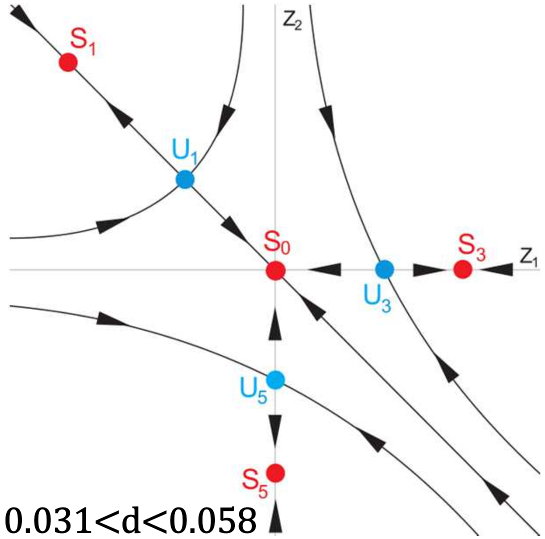
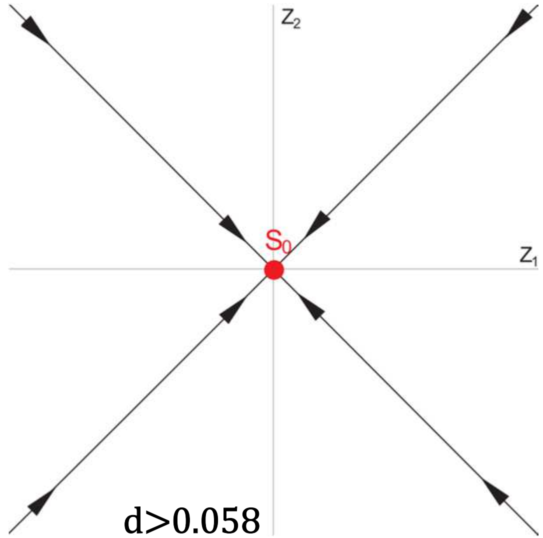




$\alpha = 5.0, \quad \beta = 0.4$





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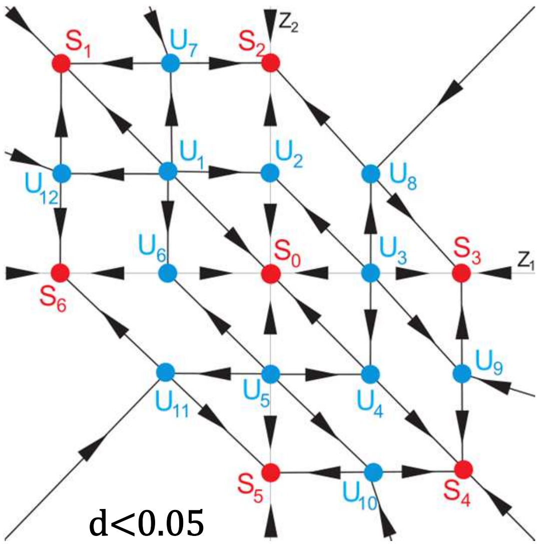
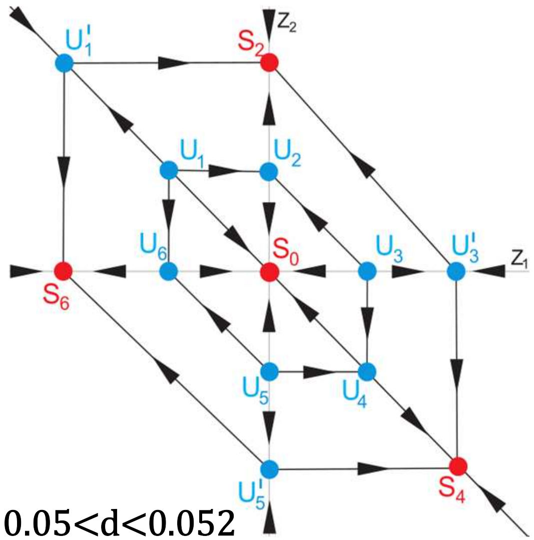
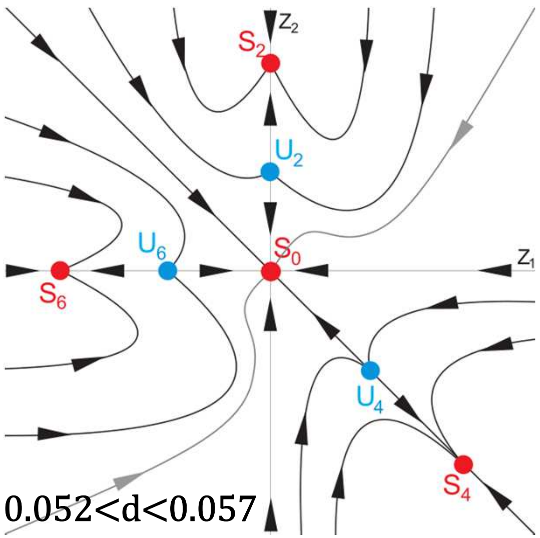
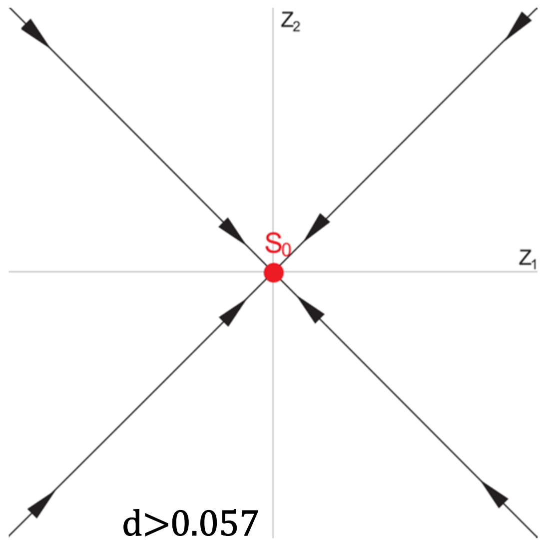




$\alpha = 4.0, \quad \beta = 2.3$





Thanks for your attention!